Observation and Reduction of Occultations of Stars by the Moon

WITH A DETERMINATION OF THE RESULTING LONGITUDE OF THE FLOWER OBSERVATORY, AND CORRECTIONS TO THE RIGHT ASCENSION, DECLINATION AND SEMI-DIAMETER OF THE MOON

BY

KRIKORIS GARABED BOHJELIAN

A THESIS

PRESENTED TO THE FACULTY OF THE GRADUATE SCHOOL OF THE UNIVERSITY OF PENNSYLVANIA IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

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INTRODUCTION.

It is well known that corrections to the coördinates, distance, and size of the Moon can be determined from the observations of occultations more accurately than from any other method.

If simultaneous observations of this kind are secured from two stations on the Earth, which differ widely in latitude, the oblateness of the Earth can also be found; and whatever the situation of the stations, their difference in longitude can be thus determined with a higher accuracy than by any other method, except that of the Telegraph and Wireless.

As the Longitude of the Flower Observatory was accurately determined, both by Telegraph and Wireless method, a comparison of these results with a value found from occultations becomes of interest, and as the later observations have a special value for improving our knowledge of the Moon's motion, the following piece of work was undertaken with these objects.

The work was begun in the early summer of 1914, the observations being made with the 18-inch equatorial of the Flower Observatory of the University of Pennsylvania.

Among the occultations observed it was learned that 13 had been simultaneously observed by Prof. Asaph Hall, with the 26-inch equatorial of the U. S. Naval Observatory.

Through the courtesy of the Director, Captain J. A. Hoogewerff, these observations were forwarded to Prof. Eric Doolittle.

It is the results from these stars, which form the principal basis of the following investigation.

OBSERVED TIMES OF OCCULTATIONS.

Date, 1914	Star	Phase	Phila. M. T.	Washington M. T.
July 17	q Tauri	I	$13^h\ 59^m\ 33^s.4$	$13^h\ 51^m\ 6^s.0$
" "	20 "	I	14 11 18 .0	14 2 33 .6
" "	16 "	E	14 38 57 .8	14 29 4 .1
" "	q "	E	14 57 4 .2	14 47 37 .5
" "	20 "	E	15 6 57 .8	14 57 47 .1
" "	21 "	E	15 19 3 .2	
" "	22 "	E	15 22 23 .2	
Aug. 7	λ Aquarii	I	11 7 52 .9	10 57 23 .6
" "	"	E	12 28 19 .9	12 16 25 .6
" "	78 "	I	12 46 42 .4	12 34 35 .3
" "	78 "	E	14 7 2 .7	13 55 52 .4
" 30	τ Sagit	I	9 14 18 .2	9 3 50 .6
" "	" "	E	10 30 18 .9	10 20 53 .3
Sept. 14	κ Gemin.	I	14 16 3 .2	14 8 20 .9
" "	"	E	14 50 0 .4	14 39 30 .0

Preliminary Computation.

The right ascension and declination of the above stars, reduced to apparent place for the observed times are as follows:

Date, 1914	Name	Right Ascension, α	Declination, δ
July 17	q Tauri	55° 1′ 39″.45	+24° 12′ 4″.72
" "	20 "	55 10 58 .50	24 6 9. 90
" "	16 "	54 55 42 .11	24 1 21 .47
" "	21 "	55 12 5 .68	24 17 23 .16
" "	22 "	55 14 12 .27	+24 15 47 .93
Aug. 7	λ Aquarii	342 2 49 .65	− 8 1 56 .12
" "	78 "	342 32 15 .38	− 7 39 24 .26
" 30	τ Sagittarii	285 24 35 .99	−27 47 53 .78
Sept. 14	κ Geminorum	114 49 28 .89	+24 36 19 .74

The right ascension, declination and horizontal parallax of the Moon for consecutive hours on the successive dates are found to have the following values:

Date, 1914		A	D	π
July 17	18^h	53° 29′ 14″.70	+24° 27′ 24″.0	54′ 48″.98
	19	54 1 31 .05	34 59 .8	50 .03
	20	54 33 52 .45	42 29 .0	51 .10
	21	55 6 19 .50	49 51 .6	52 .17
	22	55 38 51 .75	+24 57 7 .4	54 53 .26
Aug. 7	15	340 49 21 .90	− 7 45 44 .9	55 28 .89
	16	341 17 20 .40	7 31 24 .3	27 .56
	17	341 45 15 .60	7 17 2 .7	26 .28
	18	342 13 7 .65	7 2 40 .1	25 .01
	19	342 40 56 .70	− 6 48 16 .5	55 23 .74
Aug. 30	13	284 30 16 .95	−27 1 29 .5	57 25 .78
	14	285 7 12 .30	26 56 14 .7	24 .47
	15	285 44 2 .55	26 50 50 .6	23 .15
	16	286 20 47 .70	−26 45 17 .2	57 21 .83
Sept. 14	18	113 7 15 .90	+25 31 35 .6	56 58 .49
	19	113 42 37 .05	24 18 .4	57 0 .71
	20	114 17 56 .85	16 52 .1	57 2 .94
	21	114 53 15 .00	+25 9 16 .8	57 5 .18

The coördinates of the Moon's center and their derivatives for the above hours are computed from the formulæ:

$$x = \frac{\cos D \sin (A - \alpha)}{\sin \pi},$$

$$y = \frac{\sin (D - \delta) \cos^2 \tfrac{1}{2}(A - \alpha) + \sin (D - \delta) \sin^2 \tfrac{1}{2}(A - \alpha)}{\sin \pi}.$$

Date		x	x'	y	y'
July 17					
q Tauri	18^h	−1.5344714	+.5370561	+0.2879661	+.1335941
	19	− .9973448	1910	.4215411	572
	20	− .4601054	2817	.5550831	282
	21	+ .0772044	3309	.6886000	.1335060
20 Tauri	18	−1.6891416	2214	.3976049	.1329862
	19	−1.1518438	3673	.5305682	409
	20	−0.6144210	4714	.6634894	031
	21	+0.0769149	.5375338	.7663772	.1328740
16 Tauri	18	−1.4356003	.5369492	.4824448	.1339496
	19	−0.8985849	.5370746	.6163670	8963
	20	−0.3614650	1081	.7502065	8517
	21	+0.1757173	2065	.8840731	.1338142
21 Tauri	19	−1.1704081	4028	.3261183	.1328847
	20	−0.6329631	4872	.4590122	8943
	21	−0.0954338	5714	.5918895	8515
	22	+0.4421798	6558	.7246927	7562
22 Tauri	19	−1.2053922	4409	.3553572	7474
	20	−0.6679063	5296	.4881108	7515
	21	−0.1303335	6169	.6208439	7064
	22	+0.4073266	.5377030	.7535066	.1326120
Aug. 7					
λ Aquar.	15	−1.3119302	.4987673	.2898113	.2599905
	16	−0.8131459	7982	.5497641	9144
	17	−0.3143401	8103	.8096338	8292
	18	−0.1844686	8039	1.0694310	.2597498
78 Aquar.	16	−1.3391549	2319	0.1422995	.2602937
	17	−0.8408942	2868	.4025699	2468
	18	−0.3425869	3251	.6627924	1979
	19	+0.1557504	.4983468	.9229651	.2601472
Aug. 30					
τ Sagit.	13	−0.8425482	.5723855	.8049339	.0945414
	14	−0.2701396	4281	.8994167	4283
	15	+0.3023008	4491	.9938027	3518
	16	+0.8747514	.5724485	1.0881326	+.0943142
Sept. 14					
κ Gemin.	18	−1.6188750	.5594691	.9800360	−.1344901
	19	−1.0593263	5259	.8458102	2051
	20	−0.4998271	5686	.7115758	2730
	21	+0.0597563	+.5595951	+0.5772448	−.1343938

The coördinates and their derivatives for Philadelphia and Washington are computed from the formulæ:

$$\rho \sin \varphi' = b \sin B \qquad \xi = \rho \cos \varphi' \sin (\mu - \alpha)$$

$$\rho \cos \varphi' \cos (\mu - \alpha) = b \cos B \qquad \eta = b \sin (B - \delta)$$

where for Phila. $\varphi' = 39° 46' 32''.9$ and $\log \rho = 9.999400$

$$\text{long.} = 5^h\ 1^m\ 6^s.51$$

and for Wash. $\varphi' = 38° 55' 14''.0$ and $\log \rho = 9.999431$ and long. $= 5^h\ 8^m\ 15^s.78$.

		I		E	
		ξ	η	ξ	η
q Tauri	Phila.	−.7674918	+.5816687	−.7427171	+.5034517
	Wash.	−.7785898	+.5811267	.7634340	.5062870
20 "	Phila.	.7664782	.5669897	.7342088	.4918996
	Wash.	.7790062	.5666500	.7540638	.4904268
16 "	Phila.	.7556986	.5289977		
	Wash.	.7722021	.5287325		
21 "	Phila.			.7215053	.4747025
22 "	Phila.			.7180547	.4711018
λ Aqua.	Phila.	.4707500	.7173353	.83796466	.7345912
	Wash.	.5200407	.6997824	.2926039	.7196153
78 "	Phila.	.1979788	.7320044	−.0692538	.7350480
	Wash.	−.2406685	.7292446	−.0323282	.7230056
τ Sagit.	Phila.	+.1569340	.9155368	+.3904763	.8737904
	Wash.	+.1214520	.9116275	+.3611283	.8746794
κ Gemin.	Phila.	−.7668061	.5673483	−.7535621	.5202768
	Wash.	−.7790125	+.5643084	−.7705400	+.5202708

With the assumed value of the longitude of Flower Observatory viz: $5^h\ 8^m\ 6^s.51$ and of the U. S. Naval Observatory viz: $5^h\ 8^m\ 15^s.78$, we reduce Phila. and Wash. times to Greenwich times and assuming the values of t' sufficiently near to these times, so that x and y may be assumed to vary uniformly during the interval we compute certain auxiliaries first introduced by Bessel by the formulæ:

$$m \sin M = x_0 - \xi \qquad n \sin N = x'$$

$$m \cos M = y_0 - \eta \qquad n \cos N = y'$$

$$\sin \psi = \frac{m}{k} \sin (M - N) \quad \log k\ 9.4353760.$$

After obtaining the above auxiliaries viz: $\log m$, $\log n$, M, N,

and ψ we next compute Ω, T, $\bar{x}$, v, from the formulæ:

$$\Omega = h\left[\frac{k}{n}\cos\psi - \frac{m}{n}\cos(M-N)\right] - (t'-t) \quad h = 3600$$

$$T = t - \frac{1}{n}(x_0 \sin N - y_0 \cos N)$$

$$\bar{x} = -x_0 \cos N - y_0 \sin N \qquad v = \frac{h}{n\pi}$$

		Ω		T	$\log \bar{x}$	v
		Phila.	Wash.			
q Tauri	I	$5^h\ 1^m\ 19^s.79$	$5^h\ 8^m\ 41^s.89$	+20.565	9.81273	+1.98
	E	19 .47	35.82	20.565	9.81269	"
20 "	I	19 .00	33 .84	20.790	9.89853	"
	E	19 .82	42 .42	20.790	9.89849	"
16 "	E	19 .53	40 .11	20.315	9.91135	"
21 "	E	19 .19		20.911	9.77634	"
22 "	E	20 .28		20.960	9.80208	"
λ Aquar.	I	19 .13	32 .51	16.831	9.93616	1.92
	E	17 .68	36 .43	17.762	0.05258	"
78 "	I	18 .99	15 .63	17.995	9.87278	"
	E	18 .72	41 .60	17.995	9.87080	"
τ Sagit.	I	18 .90	37 .78	14.207	9.96912	1.80
	E	12 .67	31 .96	14.208	9.96912	"
κ Gemin.	I	23 .94	32 .74	20.446	0.00112	1.82
	E	5 1 23 .85	5 8 32 .49	+20.506	9.93839	+1.82

The coefficients for the final equations are obtained from the expressions:

$$v \tan\psi, \quad v.E = v(n(t + W - T) - \tan\psi x), \quad v \sec\psi$$

		Philadelphia			Washington		
		$v \tan\psi$	$v \cdot E$	$v \sec\psi$	$v \tan\psi$	$v \cdot E$	$v \sec\psi$
q Tauri	I	+0.779	−2.200	−2.13	+0.779	−2.240	−2.130
	E	+0.131	−0.732	+1.98	−0.188	−0.815	+1.980
20 "	I	+0.423	−2.140	−2.02	+0.385	−2.070	−2.02
	E	−1.160	+0.207	+2.29	−1.120	+0.134	+2.27
16 "	E	−0.963	+0.073	+2.20	−0.926	+0.004	+2.18
21 "	E	+0.267	−0.782	+1.99			
22 "	E	−0.033	−0.595	+1.98			
λ Aquari	I	+0.068	−0.751	−1.92	−0.017	−0.811	−1.92
	E	−0.732	−0.092	+4.06	−0.673	+1.210	+2.04
78 "	I	+0.040	−0.240	−1.93	+0.083	−0.365	−1.94
	E	−1.010	+1.990	+2.18	−0.946	+1.870	+2.05
τ Sagit.	I	+0.361	−0.271	−1.84	+0.342	−0.335	−1.83
	E	−1.010	+2.320	+2.06	−0.958	+2.230	+2.04
κ Gemin.	I	+1.960	−3.180	−2.67	+2.090	−3.330	−2.77
	E	−3.880	+2.970	+4.28	−4.470	+3.130	+4.82

FORMATION OF THE FINAL EQUATIONS OF CONDITIONS.

Writing the results thus far obtained, we may now set up the following equations, which we divide into four groups:

July 17. Group I′ 1

$W = 5^h\ 1^m\ 19^s.79 - 1.98\gamma + 0.779\vartheta - 2.13\pi\Delta\kappa - 2.200\Delta\pi$ (2)
$W = \quad\ 19.49 - 1.98 + 0.131 + 1.98 - 0.732$ (4)
$W = \quad\ 19.00 - 1.98 + 0.423 - 2.02 - 2.140$ (6)
$W = \quad\ 19.82 - 1.98 - 1.160 + 2.29 + 0.207$ (8)
$W = \quad\ 19.53 - 1.98 - 0.963 + 2.20 + 0.073$ (10)
$W = \quad\ 19.19 - 1.98 + 0.267 + 1.99 - 0.782$ (30)
$W = 5\ 1\ 20.28 - 1.98 - 0.033 + 1.98 - 0.595$ (40)
$W' = 5\ 8\ 41.89 - 1.98 + 0.799 - 2.13 - 2.240$ (1)
$W' = \quad\ 35.83 - 1.98 + 0.188 + 1.99 - 0.815$ (3)
$W' = \quad\ 33.84 - 1.98 + 0.385 - 2.01 - 2.070$ (5)
$W' = \quad\ 42.42 - 1.98 - 1.120 + 0.27 + 0.134$ (7)
$W' = 5\ 8\ 40.11 - 1.98 - 0.926 + 2.18 + 0.004$ (9)

Aug. 7. Group II′ 1

$W = 5^h\ 1^m\ 19^s.13 - 1.92\gamma + 0.068\vartheta - 1.93\pi\Delta\kappa - 0.751\Delta\pi$ (12)
$W = \quad\ 17.69 - 1.92 + 0.732 + 2.06 - 0.092$ (14)
$W = \quad\ 18.99 - 1.92 + 0.040 - 1.93 - 0.240$ (16)
$W = 5\ 1\ 18.72 - 1.92 - 1.010 + 2.18 + 1.990$ (18)
$W' = 5\ 8\ 32.51 - 1.92 + 0.017 - 1.92 - 0.811$ (11)
$W' = \quad\ 36.43 - 1.92 - 0.673 + 2.04 + 1.210$ (13)
$W' = \quad\ 15.63 - 1.92 + 0.084 - 1.94 - 0.365$ (15)
$W' = 5\ 8\ 41.60 - 1.92 - 0.946 - 2.05 + 1.870$ (17)

Aug. 30. Group III′ 1

$W = 5^h\ 1^m\ 18^s.90 - 1.80\gamma + 0.361\vartheta - 1.84\pi\Delta\kappa - 0.271\Delta\pi$ (20)
$W = 5\ 1\ 12.67 - 1.80 - 1.010 + 4.06 + 2.320$ (22)
$W' = 5\ 8\ 37.78 - 1.80 + 0.342 - 1.83 - 0.335$ (19)
$W' = 5\ 8\ 31.96 - 1.80 - 0.958 + 2.04 + 2.230$ (21)

Sept. 14. Group IV′ 1

$W = 5^h\ 1^m\ 23^s.94 - 1.82\gamma + 1.96\vartheta - 2.67\pi\Delta\kappa - 3.18\Delta\pi$ (24)
$W = 5\ 1\ 23.85 - 1.82 - 3.88 + 4.28 + 2.97$ (26)
$W' = 5\ 8\ 32.74 - 1.82 + 2.09 - 2.77 - 3.33$ (23)
$W' = 5\ 8\ 32.49 - 1.82 - 4.47 + 4.82 + 3.13$ (25)

If we assume γ, ϑ, $\Delta\pi$, and $\pi\Delta k$ to be the same in all of these four groups—an assumption which involves no appreciable error—we shall have 26 equations in four groups, between those quantities and w and w'. The longitude of Washington ($5^h\ 8^m\ 15^s.78$) however, will be considered to be correctly obtained.

It is evident, however, that for various reasons a direct solution of these equations in each group will not be expedient. In the first place, the large terms involved would render the operation very laborious, and furthermore, it will not be possible to separate $\Delta\pi$ from the remaining quantities, without assuming both w and w' to be known.

We therefore proceed as follows: Assuming the equations of equal weight, we subtract the first from the third, the third from the fifth, etc.; and the fourth from the second, the sixth from the fourth, etc.; continuing thus we obtain the following groups of equations:

Group I′ 2

$$
\begin{array}{rrrrr}
+0.648\vartheta & -4.110\pi\Delta\kappa & = & +1.470\Delta\pi & -0.32 \\
+0.292 & -3.000 & = & +1.410 & +0.47 \\
+1.580 & -4.310 & = & +2.350 & +0.82 \\
+0.195 & -0.092 & = & +0.134 & +0.29 \\
-1.740 & +4.320 & = & -2.280 & +0.26 \\
+0.611 & -4.120 & = & +1.420 & -6.07 \\
+0.197 & -4.000 & = & +1.850 & +1.98 \\
+1.510 & -4.290 & = & +2.200 & +8.58 \\
+0.195 & -0.089 & = & +0.130 & +2.31 \\
+1.730 & +4.320 & = & -2.210 & +1.78
\end{array}
$$

Group II′ 2

$$
\begin{array}{rrrrr}
+0.800\vartheta & +3.99\pi\Delta\kappa & = & +0.659\Delta\pi & -1.44 \\
-0.772 & +3.99 & = & +0.148 & +1.30 \\
+1.050 & -4.10 & = & +2.230 & -0.27 \\
+1.080 & +4.10 & = & -2.740 & +0.41 \\
+0.690 & -3.96 & = & +2.020 & +3.92 \\
-0.756 & +3.98 & = & -1.580 & -20.80 \\
+1.030 & -3.99 & = & +2.240 & +25.90 \\
-0.963 & +2.97 & = & -2.680 & -9.09
\end{array}
$$

Group III′ 2

$$
\begin{array}{rrrrr}
+1.37\vartheta & -3.90\pi\Delta\kappa & = & +2.59\Delta\pi & -6.23 \\
+1.30 & -3.88 & = & +2.56 & -5.82
\end{array}
$$

Group IV′ 2

$$
\begin{array}{rrrrr}
+5.84\vartheta & -6.96\pi\Delta\kappa & = & +6.15\Delta\pi & -0.09 \\
+6.56 & -7.60 & = & +6.47 & -0.25
\end{array}
$$

By means of these four groups of equations of condition, viz: (I′2, II′2, III′2, IV′2) we now determine the most probable values of ϑ and $\Delta\pi k$. The value of $\Delta\pi$, however, cannot be well determined, as we have before remarked, If it were not

known *a priori* that such was the case, it would be shown from the normal equations, which would be practically indeterminate for this quantity.

We should, therefore, determine ϑ and $\pi\Delta k$ in terms of $\Delta\pi$, in order to see what effect an error in π will have upon the longitude.

We derive from the above equations, for groups I′2 and II′2, only, the following two sets of normal equations; the last two groups are solved as they stand, since there are two equations in each group.

Normal to Group I′ 2 (or I′ 3)

$$+ 11.81\vartheta - 21.23\pi\Delta\kappa = + 10.07\Delta\pi + 14.50$$
$$- 21.23\vartheta + 133.05\pi\Delta\kappa = - 62.51\Delta\pi - 14.89$$

Normal to Group II′ 2 (or II′ 3)

$$+ 6.54\vartheta - 12.4\pi\Delta\kappa = + 7.27\Delta\pi + 51.9$$
$$- 12.40\vartheta + 128.6\pi\Delta\kappa = - 48.30\Delta\pi - 186.6$$

From I′3 we obtain

$$\vartheta = + 1.428 + .008\Delta\pi$$

$$\pi\Delta k = + 0.112 - .470\Delta\pi$$

To find γ we now substitute these values in (1), (3), (5), (7), and (9), and observing that $w' = 5^h\ 8^m\ 15^s.78$, we find the mean value of γ to be

$$\gamma = + 6''.52 - .639\Delta\pi$$

We now substitute these values of ϑ, $\pi\Delta k$, γ in (2), (4), (6), (8), (10), (30), (40), when we find the following values for the difference of longitude between Greenwich and the Flower Astronomical Observatory of the University of Pennsylvania:

$$w = 5^h\ 1^m\ 7^s.77 + .07\Delta\pi$$
$$w = 5\ 1\ 6.99 - .40$$
$$w = 5\ 1\ 6.49 - .07$$
$$w = 5\ 1\ 6.53 + .39$$
$$w = 5\ 1\ 6.51 + .29$$
$$w = 5\ 1\ 6.89 - .48$$
$$w = 5\ 1\ 7.54 - .27$$
$$\text{Mean}\quad w = 5^h\ 1^m\ 6^s.96 - .06\Delta\pi$$

And the resulting longitude from Washington is

$$\lambda = -7^m\ 8^s.82 - .06\Delta\pi$$

With the above values of γ and ϑ we may now determine corrections to the assumed right ascension and declination of the Moon.

We have the formulæ:

$$\sin N \cos D \cdot \Delta\alpha + \cos N.\ \Delta\delta = \gamma$$

$$-\cos N \cos D \cdot \Delta\alpha - \sin N.\ \Delta\delta = \vartheta$$

and from these

$$\Delta\alpha = +6''.58 \qquad \Delta\delta = +2''.96$$

Assuming the errors of the star places to be inappreciable, these will represent the errors in the computed right ascension and declination of the Moon at a time corresponding to the mean of times of the observations.

These corrections, it will be seen, are affected by any small outstanding error in the parallax, as they have been derived by assuming $\Delta\pi = 0$.

In the same way, assuming $\Delta\pi = 0$ and taking the mean of the values given above, viz: 3291″ we find from the above value of $\pi\Delta k = +.''112$

$$\Delta k = +.000034$$

we have assumed

$$k = +.272506$$

therefore,

$$k = +.272540$$

as shown from these observations.

In the same way by solving the other groups of equations, we obtain the following results:

Group	$\pi\Delta k$	ϑ	γ	$\Delta\alpha$
I	$+''.112-0.470\Delta\pi$	$+1''.428+00.008\Delta\pi$	$+6''.52-.639\Delta\pi$	$+6''.58$
II	$-.839-0.328$	$+6.350+00.500$	$+7.47+.320$	$+3.72$
III	$-.500-0.560$	$-5.960+00.300$	$+11.59+.440$	$+13.92$
IV	$-.322-9.700$	$-3.820-10.500$	$+9.93+.380$	$+11.01$

Group	$\Delta\delta$	k	w	λ
I	+2.96	.272540	$5^h\ 1^m\ 6^s.96-0.06\Delta\pi$	$-7^m\ 8^s.82-0.06\Delta\pi$
II	+9.08	.272408	5 1 6.92−0.49	−7 8.86−0.49
III	−3.99	.272386	5 1 6.86−0.80	−7 8.92−0.80
IV	−1.22	.272415	5 1 6.89−1.48$\Delta\pi$	−7 8.89−1.48$\Delta\pi$
Mean		.272437	$5^h\ 1^m\ 6^s.91-0.43\Delta\pi$	$-7^m\ 8^s.87-0.43\Delta\pi$

CONCLUSION.

The errors $\Delta\alpha$ and $\Delta\delta$ in the Moon's position are somewhat smaller than was to be expected, and indicate that this body is following its computed path somewhat more closely than in recent years.

The corrections Δk to the apparent semi-diameter is markedly negative, but it is possible that values of this quantity secured from occultations may be influenced by the aperture of the instrument employed.

The final mean value of longitude of Flower Observatory from U. S. Naval Observatory, as shown above, from this work is

$$\lambda = -7^{m}\ 8^{s}.87$$

The results previously obtained for the same quantity are:

By Telegraph $\lambda = -7^{m}\ 8^{s}.91$

By Wireless $\lambda = -7^{m}\ 8^{s}.74$